BRIEF REPORT OF
MINOR RESEARCH PROJECT

TOPIC

LATTICE TRANSLATIONS AND MOTIONS

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-- By --

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1. BRIEF REPORT

In this we prove that order of an ℓ- subgroup of a finite ℓ- group divies it'r order. Consequently we obtained that order of a convex sublattice which is $I \cap D$ divides the order of a finite lattice, where I denotes ideal and D the dual ideal. We start with dilation, translation and double dialational and state their properties.

The ℓ- group is a system which is a group and at the same time a lattice. The ℓ-dilation is defined on ℓ- group G as the function $(d_a)(x) = a \wedge x$ and translation is defined by $(t_a)(x) = x \vee a$.

Then $d_a \vee d_b = d_{a \vee b}$; $d_a \wedge d_b = d_{a \wedge b}$

and $t_a \vee t_b = t_{a \vee b}$; $t_a \wedge t_b = t_{a \wedge b}$

The joint action of translation and dilation is constant. These facts are discussed in section 2. In the same section we prove that every ℓ- group $(G, \square, \vee \wedge)$ is distributive lattic. It is also proved that dilation maps translations into translations and a transplation maps dilations into dilations. Arthur Knoebel in [7] defined a 2 - place function $f_b : B^2 \longrightarrow B$ as $f_b(x_1, x_2) = (b \wedge x) \vee (b \wedge x_2)$

Section 2 include the basic material which is required in sections -3 and 4

In any ℓ- group G

(i) $(t_a \square d_{a \wedge b} \square t_b) = a$

(ii) $(t_a \square t_{a \wedge b} \square d_b) = a$

(iii) $(t_a \square t_{a \vee b} \square t_b)(C) = a \vee b$

(iv) $(d_a \square t_{a \wedge b} \square d_b) = a \wedge b$

By combining dilation and translation we have obtained that

$(t_{a \wedge b} \square d_{a \vee b})(c) = (a \wedge b) \vee (a \wedge c) \vee (b \wedge c)$

Which is called median ternary operation.

In secon 2 dilational and double dilational hulls are introduced in an ℓ- group G . For median algebra and conves hull one can refer [2] and [3]. In [8] we find some good results on convex ℓ- subgroups of an ℓ- group. Out of these the basic results are Lemma 3.2 and Theorem 3.1 in which proved that every convex ℓ- subgroup of an ℓ- group is an ℓ- ideal. In theorem 3.1 it is proved that every convex ℓ- subgroup of an ℓ- group is the set theoratical intersection of an ℓ- ideal and of dual ℓ- ideal.

These two results serve our machinery in section 4 to prove that the order of an convex ℓ- subgroup divides the order of finite ℓ- group and also the order of convex sublattice divides the order of a finite lattice. In the same section 3, we prove that every ℓ- group is isomorphic to dilational hull. Moreover if G is an ℓ- group then G^2 is ismorphic to double dilational hull **XX** (G). The proof is very lengthy and hence it is divided into three parts, each part being a separate Lemma.

Section 4 points out the investigation on ℓ- ideals. For results on ideals of semilattices one can refer [9]. This section also include ℓ- congruences. There are very interesting results. For congruences and congruence lattices with a very good literature one can consult, General lattics Theory by G. Gratzer [6], pages 76, 161, 196 etc. Using ℓ- ideals and ℓ- congruences we have very successfully solved the problem of dividing the order of an ℓ- group by the order of it's ℓ- subgroup and further more dividing the order of a finite lattice by it's convex sublattice. The tool is discussed previously i sthe set intersection $I \cap D$ of ℓ- ideal and dual ℓ- ideal for ℓ- groups and ideal; dual ideal for lattices. We denote, (L,V, $\wedge$) as a lattice and (G,$\square$,V$\wedge$) as an ℓ- group whenever the terminology appear in the paper.